On the Characteristics of Cylindrical Journal Lubrication at High Values of the Eccentricity.

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The object of the investigation described in the present paper was to obtain more definite information than is at present available on the nature of the action between two lubricated surfaces when the value of the intensity of the pressure as defined by load/(area of lubricated surface) is considerably higher than that normally attained in current practice with cylindrical journals. In the bath lubrication experiments of Mr. Beauchamp Tower in 1883,* the load at seizing was about 600 lbs. per square inch of the projected area of the bearing. In recent practice with forced lubrication and special lubricants, it has been found possible to increase the load up to 2000 lbs. per square inch without seizing. In the case of worm gears, however, high efficiencies of power transmission are obtained at values of the pressure between the surfaces which are of the order of those of the limiting compressive stresses of the materials, and it has been suggested† that perfect lubrication, by means of a film of oil completely separating the surfaces, does not obtain in such cases, but that the thickness of the oil between the surfaces is of molecular dimensions, so that the interaction of these molecules with the molecules of the surfaces will determine the nature of the frictional resistance to motion which is experienced.

This condition has been termed Boundary Lubrication by Mr. W. B. Hardy.‡ The evidence in favour of the existence of boundary lubrication in the case of worm gears is that the efficiency of power transmission by such gears is very little affected by the viscosity of the lubricant, but depends appreciably on the constitution of the oil and nature of the solid surfaces. That this evidence is conclusive cannot, in the opinion of the writer, be regarded as certain, since it is conceivable that in these tests the thickness of the film and the area of it which is under pressure should vary in such a way as to produce changes in the resistance of the order of those due to variations of viscosity of the lubricant, and further, that the flow of the material of the worm wheel, which undoubtedly takes place under the extremely high pressures, may account for the observed effect of the nature of the material.

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* 'Proc. Inst. Mechanical Engineers,' 1885, p. 59.

† 'Report of Lubricants and Lubrication Inquiry Committee,' 1920, p. 26.

‡ 'Roy. Soc. Proc.,' A, vol. 100, p. 550.

For the purposes of the present investigation, direct observations of the action between the teeth of a worm gear would have been preferable, but as this appeared to be impracticable, recourse was made to cases of cylindrical lubrication. It is realised, of course, that the conditions of oil supply to the teeth of a worm gear are entirely different from those obtaining in cylindrical journal lubrication, since, in the former case, any two elements of surface are only momentarily in contact, and are both covered with lubricant before the next contact, whereas the surface of the bearing is always in action, and is dependent for its oil supply on oil brought up by the unloaded side of the journal.

It was thought, however, that if steady conditions of lubrication could be set up in cases in which the arc of contact of the oil film was small, say of the order of 10° to 20°, information might be obtained bearing on the limiting conditions, as regards thickness of film of lubricant, under which the hydrodynamical theory holds, and possibly on the transition from "perfect" to "boundary" lubrication.*

An attempt has accordingly been made to set up the conditions desired, and it has been found that, by increasing the difference between the radius of the bearing and that of the journal to an extent very much in excess of ordinary practice, it is possible to obtain steady conditions of lubrication with an arc of contact of the oil film as small as 15°, the fluid pressure in the film amounting to as much as 3·5 tons per square inch.

From the observed pressure distribution in the film, it has been found possible to predict the angular position of the point of nearest approach of the journal and bearing, and hence, by a comparatively simple calculation, to obtain from the theoretical expression for the pressure slope, the values of the eccentricity of the bearing and the mean viscosity of the lubricant. In the case of a journal 1 inch in diameter, it has been found that the ordinary equations of motion of a viscous fluid hold for flow between the inclined surfaces at a distance apart of 0·00005 inch or 1·2 μ.

In all the cases of cylindrical bearings under steady lubrication conditions which have been examined, the actual measured distribution of pressure has been found to agree fairly closely with the theoretical distribution, obtained from the integration of the equations of motion of a film of lubricant separating two surfaces of the same dimensions, attitude and eccentricity as the existing bearing and journal, the film having the same viscosity as that of the lubricant actually used.

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* Mr. Hardy uses the term complete lubrication to denote any hydrodynamical state of lubrication. The reason for the use of the term "perfect" in this paper is that Reynolds had previously used the term "complete" to distinguish the case in which the film of lubricant extends over the whole arc of the bearing from that in which it extends over a portion of it ("Theory of Lubrication," 'Phil. Trans.,' Part I, p. 62 (1886).

It was concluded, therefore, that the conditions were in all cases those of perfect lubrication, and no approximation to the hypothetical ones of boundary lubrication have yet been observed.

DESCRIPTION OF APPARATUS.

For the purpose of the experiments, the apparatus illustrated in fig. 1 was constructed. The journal, which is of tool steel, hardened and ground, is 1 inch in diameter and 3 inches long. The bearing consists of a cylindrical phosphor bronze bush, very carefully bored out and lapped, and its true internal diameter determined in the Metrology Department of the Laboratory. Considerable difficulty was experienced in securing the requisite uniformity in the internal diameter of the bush, which was essential, and it is due to the skill of Mr. A. Jenkinson, of the Mechanical Staff of the Department, that success in this respect was secured. In the case of the 1·02-inch bearing used for the majority of the experiments, it was found that at no point did the radius vary by more than 1 per cent. from the mean value.

The values of the pressure in the oil film were obtained by connecting a small hole in the centre section of the bearing to a Bourdon gauge, carried by the housing of the bearing. In estimating pressures by this method, great care was necessary to ensure that all air was extracted from the pipe connections and the tube of the pressure gauge.

As the bearing was in the form of a complete cylindrical bush, it was not thought necessary to attempt to secure the conditions of bath lubrication adopted in Tower's experiments with a half bush, the oil being fed into the bush from a reservoir communicating with the solid end of the bush under a head of about 4 inches. Further, in order to avoid the complication of a large number of pressure holes and pipe connections, all the pressures were measured from one hole, the angular distribution of pressure being obtained by rotating the bearing in its housing through a known angle. The validity of this method depended, of course, on no wear of the bearing taking place during the running, and evidence of wear was carefully sought during the experiments. It was found that, provided the apparatus was always started up under no load, and the load gradually imposed with

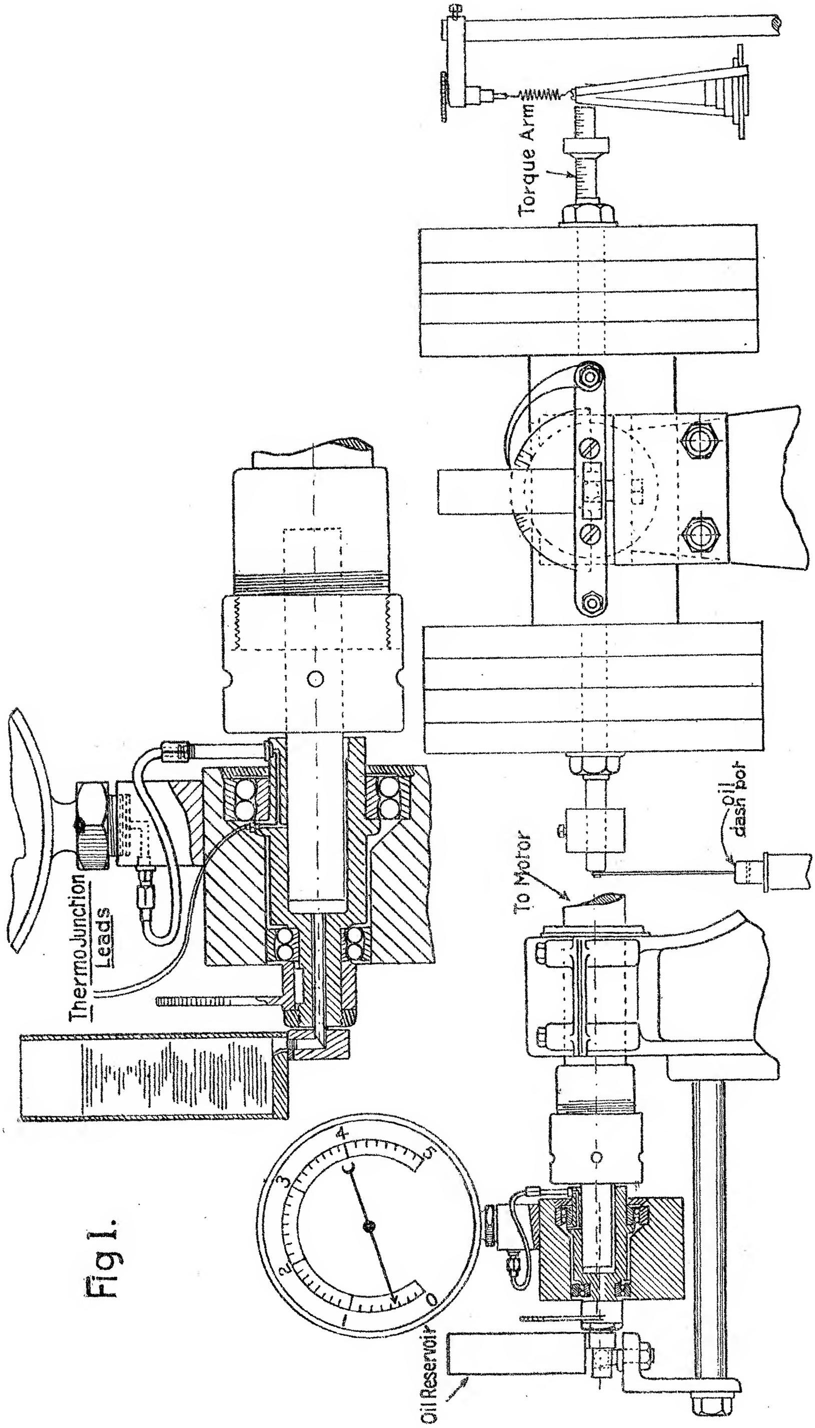

Torque Arm
To Motor
oil dash pot
Thermo Junction Leads
Oil Reservoir
0
1
2
3
4
5

Fig I.

similar precautions on stopping, no wear could be detected, even after prolonged running. It was therefore assumed that the method of rotating the bearing during a pressure distribution determination was justifiable, and obviously the saving of time due to the method was very great.

In Tower's original experiments no attempt was made to determine the temperature of the oil film, the preliminary assumption being that the value of this was that of the oil bath. It was concluded by Reynolds, in his analysis of Tower's experiments, that this assumption was not justified, and a correction was introduced based on the heat dissipated by the known frictional resistance.

In the present experiments an attempt has been made to measure the temperature of the film by inserting a thermojunction in the pressure hole, after the pressure had been determined, in such a manner that the end of the junction was in contact with the surface of the revolving journal, and a small amount of oil was allowed to leak through the oil-way by means of a special gland. It is thought that by this means the temperature of the oil film has been determined with fair accuracy, but evidence has been found that the temperature of the film is not, as would be expected, uniform throughout.

The frictional torque on the bearing was estimated by direct weighings in the scale-pan of the torque arm attached to the housing of the bearing. Want of accuracy in torque determinations, owing to the effect of static friction on the zero position of the arm was eliminated by running the journal in both directions and taking the mean reading.

RESULTS OF THE EXPERIMENTS.

Case I.—Diameter of journal, 1·00 *inch*; *diameter of bearing,* 1·02 *inch.*

The observed pressure distribution at a speed of 1000 r.p.m. and with a total load of 690 lb. is given in fig. 2 for sperm oil and in fig. 3 for rape oil. The pressure gauge readings are given in Tables I and II. The measured torques with these oils were 0·25 and 0·59 (lb.-inches), giving values of the nominal coefficient of friction of 0·00072 and 0·0017 respectively. It may be remarked that the coefficient for sperm oil is appreciably less than that found in the best ball-bearing practice.

Experiments were also made with more viscous oils, *i.e.*, castor and a mineral oil known as F.F.F. cylinder oil, but in these cases the conditions at the extremities of the oil film were evidently not stable and steady pressure gauge readings were not obtainable, whereas the fluctuations of pressure with the less viscous oils were negligible.

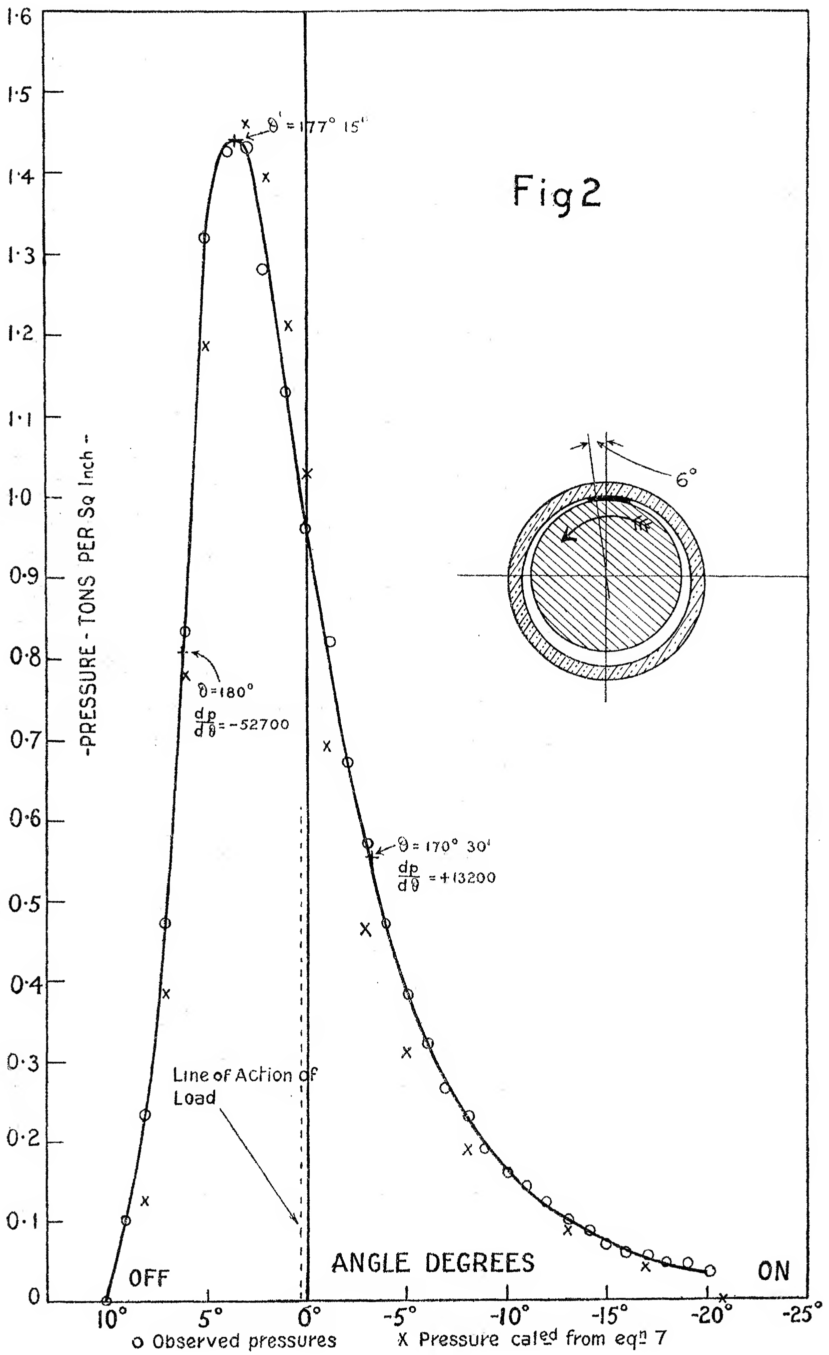

1·6
1·5
1·4
1·3
1·2
1·1
1·0
0·9
0·8
0·7
0·6
0·5
0·4
0·3
0·2
0·1
0
-PRESSURE - TONS PER Sq Inch -
θ' = 177° 15'
θ = 180°
dp/dθ = -52700
θ = 170° 30'
dp/dθ = +13200
6°
Line of Action of Load
OFF
ANGLE DEGREES
ON
10°
5°
0°
-5°
-10°
-15°
-20°
-25°
o Observed pressures
X Pressure calcd from eqn 7

Fig 2

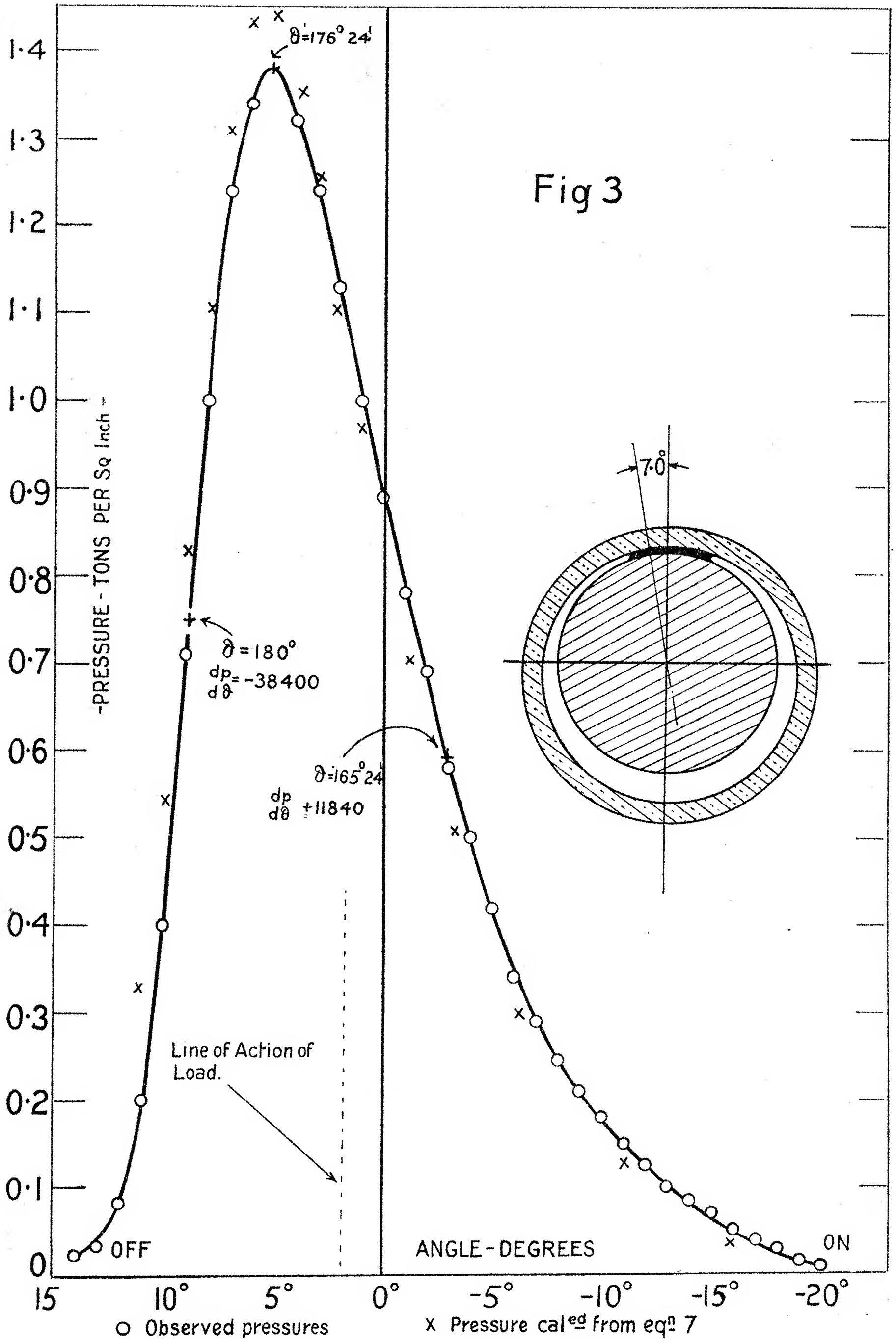


Fig 3

1·4
1·3
1·2
1·1
1·0
0·9
0·8
0·7
0·6
0·5
0·4
0·3
0·2
0·1
0
-PRESSURE - TONS PER Sq Inch -
ϑ=176° 24'
ϑ = 180°
dp/dϑ = -38400
ϑ=165° 24'
dp/dϑ ±11840
7·0°
Line of Action of Load.
OFF
ON
ANGLE - DEGREES
15
10°
5°
0°
-5°
-10°
-15°
-20°
O Observed pressures
X Pressure cal^ed from eq^n 7

Case II.—Diameter of journal, 1·00 *inch; diameter of bearing,* 1·06 *inch.*

It was thought that in the case of the more viscous oils steadier pressure conditions would probably be obtained by the use of a still greater difference in the radii of bearing and journal, and a bearing having an internal diameter of 1·06 inch was made. On making a series of observations at the same load and speed as before, it was found that this conclusion was correct, and satisfactory tests were made with castor oil and F.F.F. cylinder oil. The results obtained are shown in figs. 4 and 5 and in Tables III and IV.

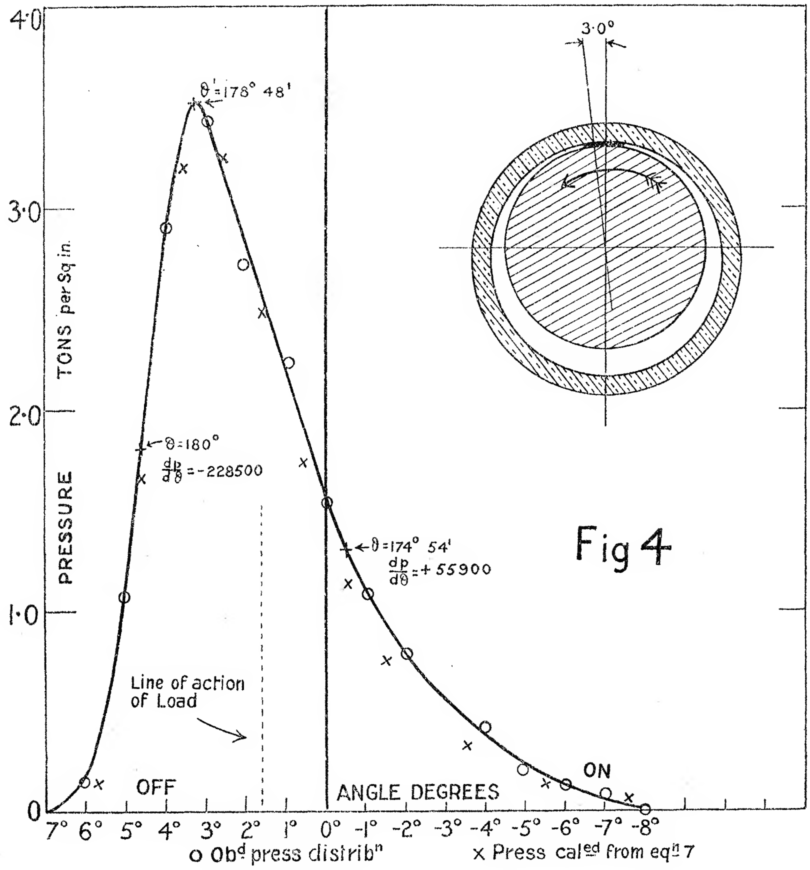
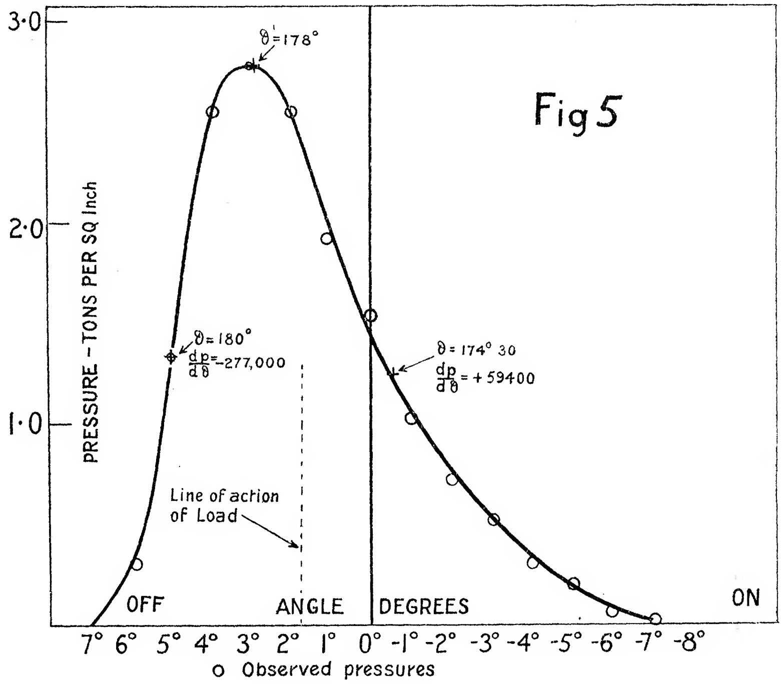


Fig 4

It will be seen that in these experiments steady conditions of lubrication were obtained with an arc of contact of the film of from 14° to 15°, and with fluid pressures up to 3·5 tons per square inch. The nominal coefficients of friction were, however, higher than in the previous cases, the values being 0·0023 for the castor oil and 0·0035 for the F.F.F. cylinder oil.

Experiments were also tried with this bearing at the same load and speed, and using sperm oil as a lubricant, but it was not found possible to attain steady conditions before "seizing" occurred, so that the limit as regards reduction of arc of contact of film with this method of supplying the lubricant appears to have been reached.



Calculation of the Eccentricity.

In fig. 6 let O be the centre of the journal of radius a, O′ the centre of the bearing of radius $a+\eta$, OA the radius through the point of nearest approach

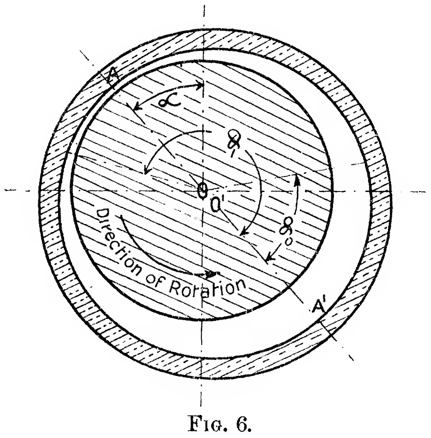


Fig. 6.

making an angle α with the vertical. Then if $OO' = c\eta$ the distance apart of the surfaces can be written

$$h = \eta\,(1+c\cos\theta), \tag{1}$$

where θ is measured from OA' in a counter-clockwise direction. c is called the eccentricity of the bearing.

Assuming the space between the surfaces from θ_0 to θ_1 to be filled with a lubricant in steady motion, it was shown by Reynolds,* from the hydrodynamical theory of a viscous fluid, that the pressure slope in the film is given by

$$\frac{dp}{d\theta} = \frac{6\mu Uac\,(\cos\theta - \cos\theta')}{\eta^2\,(1+c\cos\theta)^3}, \tag{2}$$

where θ' is the value of θ at the point of maximum pressure, μ the viscosity of the film, and U the relative velocity of the solid surfaces; and that the intensity of the frictional resistance at the surface of the bearing is

$$f = \left[\frac{\mu U}{h} - \frac{1}{2}\frac{h}{a}\cdot\frac{dp}{d\theta}\right]_{y=h}. \tag{3}$$

Assuming, therefore, that the bearing is in equilibrium under the load L, acting through O, the fluid pressure and friction, and an externally applied couple, M, per unit of length, we have

$$\int_{\theta_0}^{\theta_1}\{p\sin(\theta+\alpha) + f\cos(\theta+\alpha)\}\,d\theta = 0, \tag{4}$$

$$\int_{\theta_0}^{\theta_1}\{-p\cos(\theta+\alpha) + f\sin(\theta+\alpha)\}\,a\,d\theta = L, \tag{5}$$

$$\int_{\theta_0}^{\theta_1} fa^2\,d\theta = M, \tag{6}$$

and another equation representing the boundary conditions of the fluid.

It would appear, therefore, that, provided L, M, U, a, and μ are known, the values of η, c, α, and θ' and the distribution of pressure round the bearing could be determined.

As has been remarked by Harrison,† equation (2) is directly integrable, the solution being

$$\begin{aligned} p = c + \frac{6\mu Ua}{(1-c^2)^2\eta^2}\Bigg[&\frac{c\sin\theta}{(1+c\cos\theta)^2}\{-(1-c^2)(1+c\cos\theta) \\ &+ \tfrac{1}{2}(1+c\cos\theta')\,[1-c^2+3\,(1+c\cos\theta)]\} \\ &+ (1-c^2)^{-\frac{1}{2}}\{2(1-c^2)-(1+c\cos\theta')(2+c^2)\}\tan^{-1}\left\{\left(\frac{1-c}{1+c}\right)^{\frac{1}{2}}\tan\frac{\theta}{2}\right\}\Bigg]. \end{aligned} \tag{7}$$

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* 'Phil. Trans.,' Part I, p. 194 (1886).

† 'Trans. Cambridge Phil. Soc.,' vol. 22, No. III.

It will be apparent, therefore, that the integration of equations (4), (5), and (6) in a manner suitable for direct evaluation of η, c, α, and θ' would involve extremely laborious and complex calculations, and to avoid this Reynolds resorted to the device of the expansion of $dp/d\theta$ in a series of ascending powers of c, and then integrated term by term. The solution of equations (4), (5), and (6) was thus rendered comparatively simple, but as the series were not convergent for values of c greater than 0·6, the solution thus obtained was not applicable to cases of bearings in which the eccentricity was high. This restriction was not, however, considered by Reynolds to be an important one, for the reason that in the practicable case of a semi-cylindrical bearing he was led to the conclusion that the pressure at the off extremity of the bearing becomes negative when the value of c exceeds 0·5. Since, in his opinion, the existence of negative pressures in the film would probably lead to rupture of the film, this value of c was regarded by Reynolds as the limit of safe load, and therefore, as will be seen from equation (1), "the limiting load for which the half-bearing will run full is that which caused the least distance between the surfaces to be half the difference of the radii." *

For this reason further integration of the equations for cases in which c was greater than 0·5 was not attempted by Reynolds, although he was careful to point out that since "the rupture of the film does not take place at the point of nearest approach, the brass may still be entirely separate from the journal, and could the integrations be effected, it would be as possible to deal with this condition as with that of complete lubrication," * *i.e.*, the half-bearing running full.

As, however, will be clear from the results of the experiments described above, the rupture of the film at the off extremity of the bearing, and the consequent reduction on the arc of contact of the oil film, is by no means the limit of safe load for a bearing supplied with oil in the manner here described, so that the solution obtained by Reynolds is inapplicable to the present cases, in which the value of the eccentricity is of the order of 0·99.

As no direct solution which would enable the values of the eccentricity, attitude, and position of maximum pressure to be calculated from the dimensions and speed of the bearing, the load, frictional moment, and viscosity of the oil, appeared to be available, attention was given to the much simpler problem of utilising the observed pressure distribution curve for the purpose of obtaining the value of the eccentricity, and the subsequent comparison of the values of the calculated pressure at different points of the bearing with those actually observed.

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* 'Phil. Trans.,' Part I, p. 163 (1886).

The method of calculating the eccentricity finally adopted was as follows:—

From an inspection of the experimental pressure distribution curves, it was noted that the position of the point of inflection of the curve on the off side of the bearing which, from equation (2), will be seen to locate the point of nearest approach, could be predicted with fair accuracy.

Assuming the value of α so obtained, the values of θ and $dp/d\theta$ for the point of nearest approach, and another selected point, usually the point of inflection on the "on" side, were scaled off from the curve. Inserting these values, and that of θ' in the pressure slope, equation (2), two equations were obtained from which the values of c and μ could be calculated.

If the value of μ so determined was found to be in reasonable agreement with that deduced from the measured temperature of the film, the corresponding value of c was taken to be correct.

As an example of the method the calculation of the value of the eccentricity of the 1·02-inch bearing with sperm oil, fig. 2, may be quoted. From the curve the angular positions relative to the arbitrary zero chosen of the point of nearest approach and the maximum pressure were estimated to be $+6^\circ\ 15'$ and $+3^\circ\ 30'$, so that $\theta' = 177^\circ\ 15'$. The value of the pressure slopes at 100° and $170^\circ\ 30'$ were $-52{,}700$ and $+13{,}280$ lbs. per square inch per radian respectively, so that by substitution in (2) we have

$$c = 0{\cdot}9946.$$

$$\mu = 0{\cdot}32 \text{ (in C.G.S. units).}$$

At a temperature of 35·8° C., which was the measured temperature of the film, the value of μ for sperm oil at atmospheric pressure is 0·25, so that, assuming a rise of viscosity due to pressure of some 25 per cent., the agreement between the calculated and true values of μ is fair.

The value of $c = 0{\cdot}9946$ would appear, therefore, to be approximately correct, so that the distance apart of the surfaces at the point of nearest approach, which is $\eta(1-c)$, is, in this case,

$$0{\cdot}000054 \text{ inch or } 1{\cdot}37\ \mu\ (\mu = 0{\cdot}0001 \text{ cm.}).$$

In the case of the 1·06-inch bearing it was found that, owing, presumably, to the high intensity of the oil pressures, a slight warping of the bush took place, with the observed effect that the fluid pressures directly under the main ball bearing (see fig. 1) were appreciably higher than at other sections. The pressures plotted in figs. 4 and 5 refer to this section, and the values of c are calculated from this distribution. The film temperatures were, however, measured at the central section as before, and are, for this reason, somewhat lower in value than the temperatures at the section of maximum pressure. It is considered that this accounts for the fact that the values of μ calculated from the pressure distribution are lower than those corresponding to the observed temperatures, instead of higher, as in the case of the 1·02-inch bearing.

In the case of the 1·06-inch bearing with castor oil the value of c was found to be 0·99845, but, owing to the greater diameter of the bearing, the distance apart of the surfaces, at the point of nearest approach, was not much less than in the smaller bush, its value being

$$0{\cdot}0000465'' \text{ or } 1{\cdot}18\,\mu.$$

CALCULATION OF PRESSURE DISTRIBUTION.

From the values of c and μ obtained in the manner described above, the theoretical changes of pressure from one extremity of the film to the other may be calculated from equation (7). This has been done for the cases illustrated in figs. 2, 3, and 4, the values of the calculated pressure differences being indicated by crosses.

No marked disagreement between the observed pressure distribution and that calculated in the manner described above could exist, since the values of the slope at the maximum and at the two singular points, in each case, are identical, but it is of interest to note that the total rise of pressure in the bearing can be calculated with fair accuracy, and the comparatively slow rate of rise of pressure from the extremity of the "on" side which is found from equation (7), agrees closely with what is observed in practice.

The general results of the present investigation, together with a typical case of Reynold's analysis of Tower's experiments, may be conveniently exhibited in the following tabular manner:—

Experiment.	Diameter (inches).		Lubricant.	Load in lbs. per inch run.	Relative velocity of surfaces (f.p.s.)	Maximum pressure, lbs. per square inch.	Nominal coefficient of friction.	Estimated angular position relative to vertical through centre of journal.		Eccentricity of bearing (calculated).	Least distance apart of surfaces (inches).	Coefficient of viscosity of lubricant, C.G.S. units.	
	Journal.	Bearing.						Point of nearest approach.	Point of maximum pressure.			From observed temp. of oil film at atmospheric pressure.	Calculated from pressure distribution curve.
Tower	4·0	4·003 (calculated).	Olive	1452	15·7	625	0·0013	42·0° to left.	7·0° to left.	0·5200	0·000720		
N.P.L.	1·0	1·02	Sperm	277	4·36	3230	0·0007	6·0° to left.	3°–15′ to left.	0·9946	0·000054	0·252	0·319
N.P.L.	1·0	1·02	Rape	277	4·36	3190	0·0017	7·0° to left.	3°–24′ to left.	0·9904	0·000096	0·540	0·755
N.P.L.	1·0	1·06	Castor	277	4·36	7840	0·0023	3·0° to left.	1°–48′ to left.	0·99845	0·000046	1·43	1·14
N.P.L.	1·0	1·06	F.F.F. cylinder	277	4·36	6250	0·0035	3°–15′ to left.	1°–15′ to left.	0·9978	0·000066	2·09	1·87

Table I.—Observations on 1·02″ Bearing with Sperm Oil. Load 690 lbs. R.P.M. 1000. Torque 0·25 lbs./ins. Temperature of Film, 35·8° C.

ON SIDE.

Angle, degrees	−20	−19	−18	−17	−16	−15	−14	−13	−12	−11	−10	−9	−8	−7	−6	−5	−4	−3	−2	−1	0
Pressure, tons/sq. in.	0·033	0·04	0·045	0·052	0·06	0·07	0·085	0·10	0·12	0·14	0·16	0·19	0·23	0·265	0·32	0·38	0·47	0·57	0·67	0·82	0·96

OFF SIDE.

Angle, degrees	0	+1	+2	+3	+4	+5	+6	+7	+8	+9	+10
Pressure, tons/sq. in.	0·96	1·13	1·28	1·43	1·43	1·32	0·83	0·47	0·23	0·10	0·0

Table II.—Observations on 1·02″ Bearing with Rape Oil. Load 690 lbs. R.P.M. 1000. Torque 0·59 lbs./ins. Temperature of Film 33·4° C.

ON SIDE.

Angle, degrees	−20	−19	−18	−17	−16	−15	−14	−13	−12	−11	−10	−9	−8	−7	−6	−5	−4	−3	−2	−1	0
Pressure, tons/sq. in.	0·01	0·015	0·03	0·04	0·05	0·07	0·085	0·10	0·125	0·15	0·18	0·21	0·245	0·29	0·34	0·42	0·50	0·58	0·69	0·78	0·89

OFF SIDE.

Angle, degrees	0	+1	+2	+3	+4	+5	+6	+7	+8	+9	+10	+11	+12	+13	+14	+15
Pressure, tons/sq. in.	0·89	1·00	1·13	1·24	1·32	1·38	1·34	1·24	1·00	0·71	0·40	0·20	0·08	0·03	0·02	0·0

Table III.—Observations on Bearing 1·06″ diam.* with Castor Oil. Load 690 lbs. R.P.M. 1000. Torque 0·8 lbs./ins. Temperature of Film, 49·4° C.†

	ON SIDE.									OFF SIDE.						
Angle, degrees	−8	−7	−6	−5	−4	−3	−2	−1	0	+1	+2	+3	+4	+5	+6	+7
Pressure, tons/sq. in.	0	0·07	0·13	0·22	0·40	0·55	0·78	1·07	1·52	2·23	2·72	3·42	2·90	1·07	0·15	0·0

Table IV.—Observations on Bearing 1·06″ diam.* with F.F.F. Cylinder Oil. Load 690 lbs. R.P.M. 1000. Torque 1·2 lbs./ins. Temperature of Film, 51·6° C.†

	ON SIDE.								OFF SIDE.						
Angle, degrees	−7	−6	−5	−4	−3	−2	−1	0	+1	+2	+3	+4	+5	+6	+7
Pressure, tons/sq. in.	0	0·07	0·19	0·30	0·51	0·71	1·02	1·50	1·92	2·55	2·78	2·55	1·33	0·32	0·0

* Taken at section of bearing directly under main ball bearing.

† Taken at central section of bearing.